Field Theory of Macroeconomics

General Theory on Economic Growth in Substitutional Competition

(Makroökonomische Feldtheorie: Allgemeine Theorie des ökonomischen Wachstums in Substitutionskonkurrenz, 2011)

Field theory usually refers to a construction of the dynamics of a field, i.e. a specification of how a field changes with time or with respect to other components of the field. Usually this is done by writing a **Lagrangian** and the dynamics can be obtained by using the action principle. A convenient way of classifying a field is by its **symmetries and invariances**. This idea can apply to *every basic real-world observations*.

Noether's theorem gives a precise description of this relation. It states that each continuous symmetry of a system implies that some property of that system is conserved and vica versa. The theorem was proved by Emmy Noether and published in 1918. Noether's theorem has become a *fundamental tool* of modern theoretical physics and the calculus of variations. It is important, both because of the insight it gives into conservation laws, and also as a practical calculational tool. It allows researchers to determine the conserved quantities from the observed symmetries **of any system**.

Samples from Physics: Electrodynamics, Mechanics, Gravity, Quantumdynamics (Well, it's not A theory, it's THE theory.)

Now let's apply the *"mother of all theories"* to Economics!

$$\boxed{\mathbf{M}_{p,t}\cdot\mathbf{\Lambda}_{sp}=\pm\mathbf{H}_p\cdot\mathbf{\Pi}_{sp,t}}$$

Field Theory of Macroeconomics

The book to variational calculus of macroeconomics was published in 9-2011 in *german* language. An advanced english translation will follow this year.

This Wiki is intended to help readers into the new theoretical framework of calculating economic growth.

Links to main literature so far:

D. Peetz, H. Genreith, Neues Makromodell: Die Grenzen des Wachstums: Finanz- vs. Realwirtschaft, Die Bank, Zeitschrift für Bankpolitik und Praxis, Ausgabe 3/2011, S. 20-24.

H. Genreith, Technical Paper zum Artikel in Die Bank 3/2011, S20-24, Ifara, Institut für angewandte Risikoanalyse, Febr. 2011

H. Genreith, Makroökonomische Feldtheorie, ISBN 978-3-8423-8029-5, Books on Demand, Norderstedt, 2011

D. Peetz and H. Genreith, The financial sector and the real economy, Real-World Economics Review, issue no. 57, 6 September 2011, pp. 40-47

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Wiki to the Theory: http://www.genreith.de/**doku**

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On which Hypothesis relies the General Field Theory of Macroeconomics?

The only one you have to take into account for the general theory is the well known quantity equation.

Which means it has just only to be assumed, that MV=HP holds at least locally at any special time t_x. **Nothing more.**
The rest of the procedure is just standard field theory with variational calculus.
The outcome after variational calculus then is (among others) the globally valid non-linear differential quantity equation.
To find solutions to such highly nonlinear differential equations is not an easy task.
But in most practical cases reliable linearization can be used which leads to the equations of the Special Field Theory of Macroeconomics.
In such cases **only the rules for allowed simplifications (e.g. linearization)** have to be obeyed.

How the general theory works...

First of all we have to define the full and thus General Quantity Equation:

$$\begin{pmatrix} V_{11} & V_{12} & \dots & V_{1m} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ V_{n1} & V_{n2} & \dots & V_{nm} \end{pmatrix}\cdot\begin{pmatrix} M_1 \\ . \\ . \\ . \\ M_m \end{pmatrix}=\begin{pmatrix} P_{11} & P_{12} & \dots & P_{1m} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ P_{n1} & n_{n2} & \dots & P_{nm} \end{pmatrix}\cdot\begin{pmatrix} H_1 \\ . \\ . \\ . \\ H_m \end{pmatrix}$$

The norm of this matrices and vectors are defined now by the following summation-norm:

$$[\![X]\!]_s=\sum_n X_{n1}+\dots+\sum_n X_{nm}$$

...first splitting into the two main parts:

Now we invest Euler-Lagrange, and we define in a first step:

$\dot{q}_b:=V \wedge q_b:=P$ Now we do $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_b}-\frac{\partial L}{\partial q_b}=0$ with

$$L:=M_R V_R+M_I V_I-H_R P_R-H_I P_I+M_{RI} V_{IR}-H_{RI} P_{IR}$$

.....which results after some variational calculus in :

$$\dot{K}=d\cdot Y+[e\cdot K(interest(Y))-c\cdot K(loans(Y))]$$
$$\dot{Y}=f\cdot K(loans(Y))-g\cdot K(interests(Y))$$

leads to basis model of special theory from fundamentals

The term $e\cdot K(interest(Y))-c\cdot K(loans(Y))$ is the sum-effect of giving loans to the real economy on one hand, and to get interests in return on the other hand. The net effect now can be summarized in our net-business-rate by some sorting and definition:

$$e\cdot K(interest(Y))-c\cdot K(loans(Y))=e\cdot\alpha(t)K-c\cdot\beta(t)K$$
$$=(e\cdot\alpha-c\cdot\beta)K=:P_n(t)K$$

...which means for the simplest non-trivial linearized solution to the problem:

$\dot{Y}=-p_n K$ and $\dot{K}=p_S Y+p_n K$, which just had to be shown.

In the next step, we have to define the Quantity Equation in better useable units.

...now generalizing to splitting into any number of products:

The problem is solvable for any splitting of the whole economy by introducing some suitable metrices:

This pseudo-complex (mostly called a pseudo-riemanian) metric

$$g_{ij}=\begin{pmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & \dots & 0 & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & \dots & -1 & 0 \\ 0 & 0 & \dots & 0 & -1 \end{pmatrix}$$

gives the rule $[X_C]_p=X_C^T\cdot g_{ij}\cdot X_C=\sqrt{X_1^2-X_2^2-\dots-X_n^2}$

The pseudo-complex vector $X_C=(X_1, X_{2,\dots}, X_n)$ may have any dimension.

...which leads finally to the general quantity equation:

By some new greek-symbols for our definition of the ***p*-norm pseudo-complex vectors of any dimension** the general nonlinear QE is written as:

$$\left[\frac{d}{dt}\mathbf{M}\right]_p^2\cdot[[\![\mathbf{\Lambda}]\!]_s]_p^2=[\mathbf{H}]_p^2\cdot\left[\frac{d}{dt}[\![\mathbf{\Pi}]\!]_s\right]_p^2 \qquad \textbf{(GNL-DQE).}$$

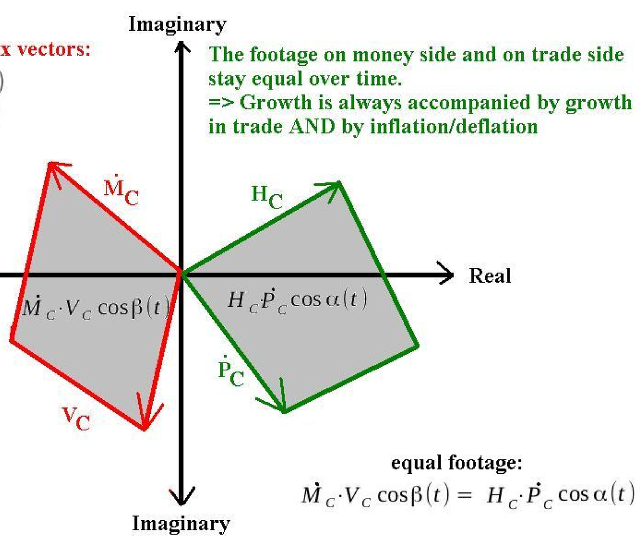
Or taking the root, $\mathbf{M}_{p,t}\cdot\mathbf{\Lambda}_{sp}=\pm\mathbf{H}_p\cdot\mathbf{\Pi}_{sp,t}$ **(NL-DQE with root applied)**

where $\mathrm{M},\mathrm{H},\Lambda$ and Π are pseudo-complex matrices with matrix-summation norm s , vector norm p and the sub-index t denotes the time-derivative respectively. The in the last equation used Einstein-notation lets look the NL-QE much like the classical QE approximation, but it is entirely of an other deeper meaning.

In general complex vectors:

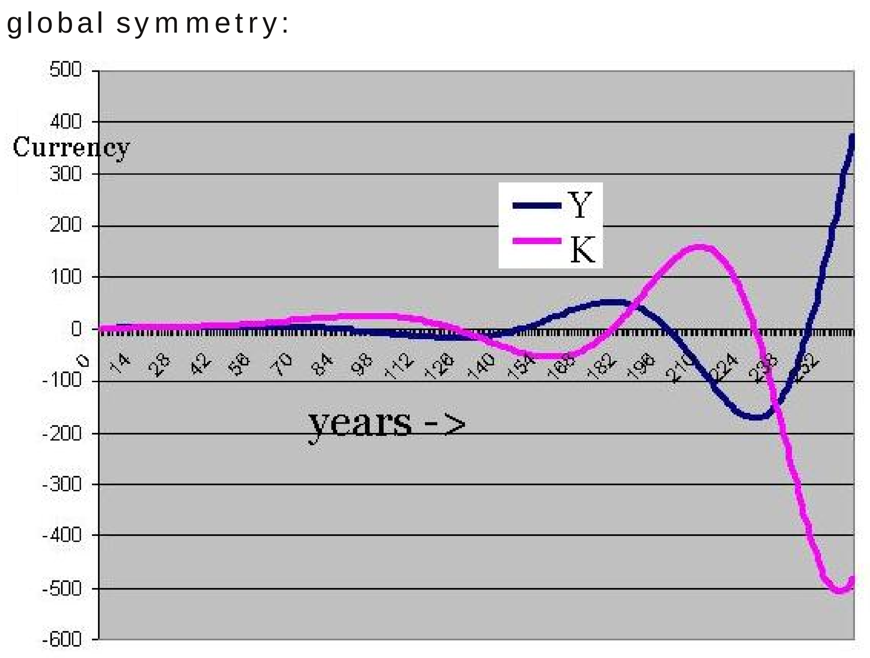
$M_C=(M_R, M_I,\dots)$
$V_C=(V_R, V_I,\dots)$
$H_C=(H_R, H_I,\dots)$
$P_C=(P_R, P_I,\dots)$

The footage on money side and on trade side stay equal over time.
=> Growth is always accompanied by growth in trade AND by inflation/deflation

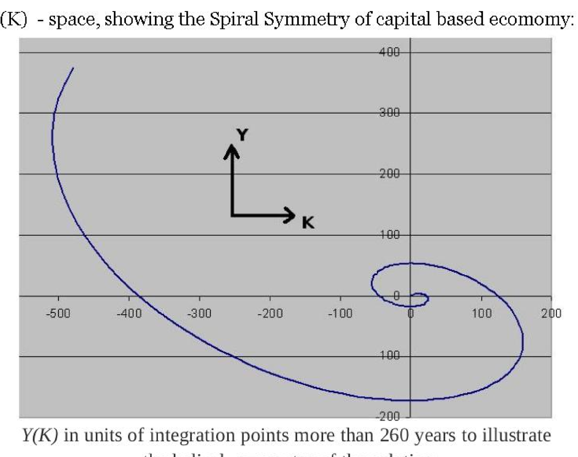


equal footage:
$\dot{M}_C\cdot V_C\cos\beta(t)=H_C\cdot\dot{P}_C\cos\alpha(t)$

Predator-Prey-Symmetry and Spiral-Symmetry: Competition by substitution means that a new product A must first have its business success at the expense of other products B. Growth is first a result of loans and at best secondary to technical approvements. This is directly related to an global symmetry:

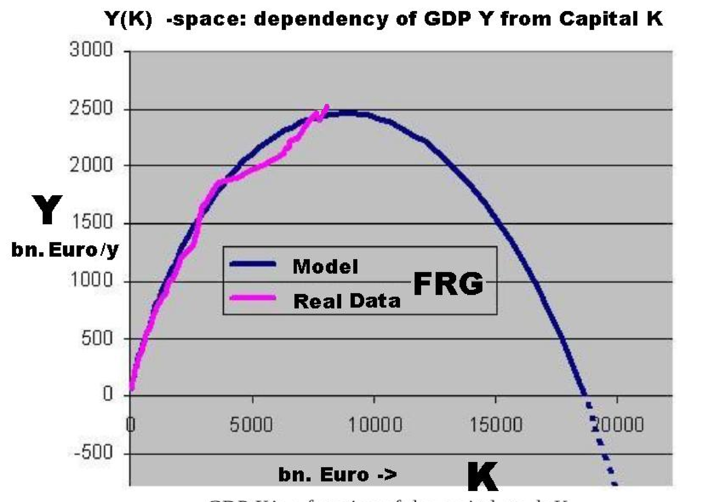


Y(K) - space, showing the Spiral Symmetry of capital based economy:



Y(K) in units of integration points more than 260 years to illustrate the helical- symmetry of the solution.

GDP Growth is limited by total capital stock:



GDP *Y* is a function of the capital stock *K*, that is, the function *Y(K)* instead of *Y(t),K(t)*, FRG in billions of € from1950 to 2010. Model data from 1950 to about 2040.

The rule of thumb , here for the FRG:

$$Y(K[Whr.])=-a_K^N K^2+b_K^N K+c_K^N \qquad [Cur./y]$$

can be determined and used for each country *N*,
The coefficients are sorted according to their meaning

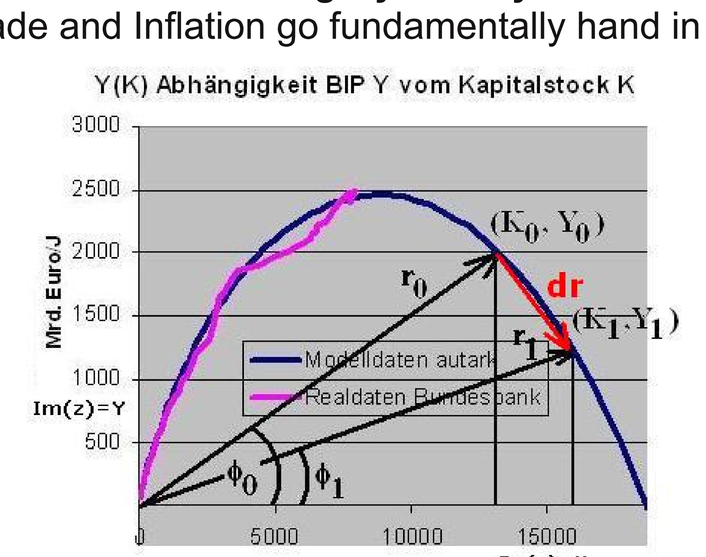
$c_K^{BRD}=197{,}9[Mrd.€/J]$	GDP-Offset ($Y_0(K_0)$)
$b_K^{BRD}=0{,}5174[1/J]$	Average Capital Efficiency
$a_K^{BRD}=2{,}852\cdot10^{-5}[1/(Mrd.€\cdot J)]$	Repressional Capital Coefficient

The maximum of the development is given by $\frac{dY}{dK}(K_{max})=0$ at $K_{max}=\frac{b_K^N}{2a_K^N}$

with $K_{max}=9071$ Billion € for the FRG. This is the maximum value of the capital raising ability of the Federal Republic until the turn of development.

Rotational-Stretching-Symmetry:

Trade and Inflation go fundamentally hand in hand



Nonlinear Solutions for Inflation/Deflation and Trade too:

$$a_I=\rho^2\exp(d\phi\frac{\psi_2}{\psi_1})\cos(\phi_0\frac{\psi_2}{\psi_1})+\frac{i}{r_0^2}\cdot\sin(\phi_0\frac{\psi_2}{\psi_1})$$

with $\psi_0:=\phi_0(\phi_1+\ln(dr))$, $\psi_1:=2(\phi_0^2-\phi_0\phi_1+\frac{\phi_1^2}{2})$, $d\phi:=\phi_0-\phi_1$

$r_0^2:=K_0^2+Y_0^2$, $r_1^2:=(K_0+1)^2+(Y_0+Y_K)^2$, $\rho^2:=r_1^2/r_0^2$ and

$\psi_2:=\psi_0-d\phi\ln(\rho^2)-\phi_1\ln(dr)$

And in accordance with this the inflation or price-formation is $I(r)=I_0(r_0)\cdot a_I$

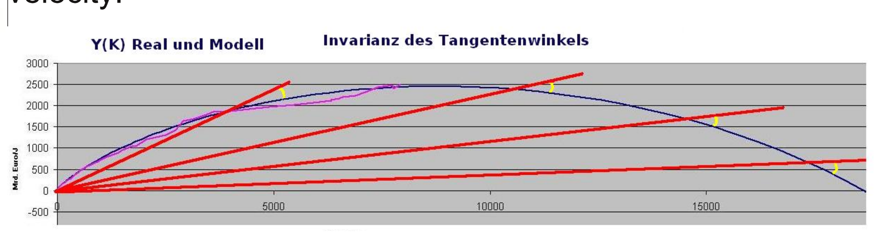
Trade Volume:

$$b_H=(\phi_1-\ln(dr)+\ln(\frac{(K^2+Y^2)}{(K^2+2K+1+Y^2+2YY_K+Y_K^2)}))\frac{(-\phi_0+i\cdot(\phi_0-\phi_1))}{(2\phi_0^2-2\phi_0\phi_1+\phi_1^2)}$$

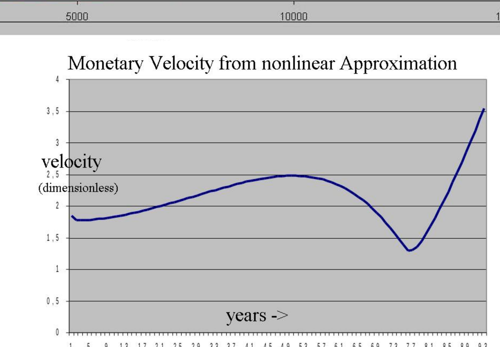
and $H(r)=H(r_0)\cdot b_H$

Symmetry: Preservation of the Tangent Angle

Makes possible to derive nonlinear equations for monetary velocity:



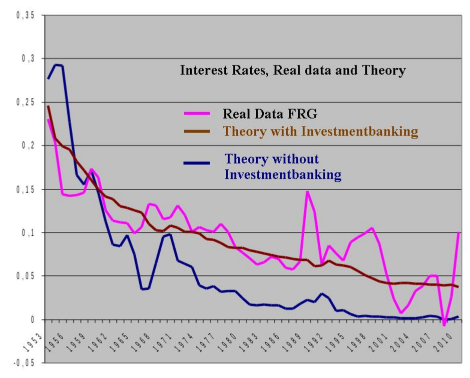
Monetary Velocity from nonlinear Approximation



The sum of the contributions up to the highest order to the above-discussed approximations $[F(n+1)+\frac{1}{2}\cdot F(n+2)+\frac{1}{6}\cdot F(n+3)]$.

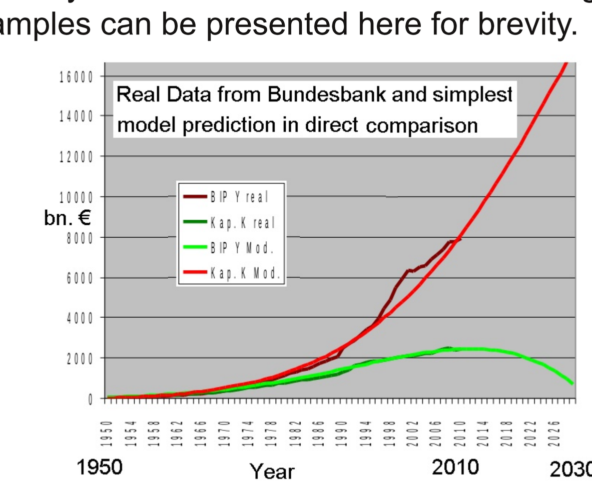
Commutator Symmetry:

Hints to calculate average interest rates of an economy

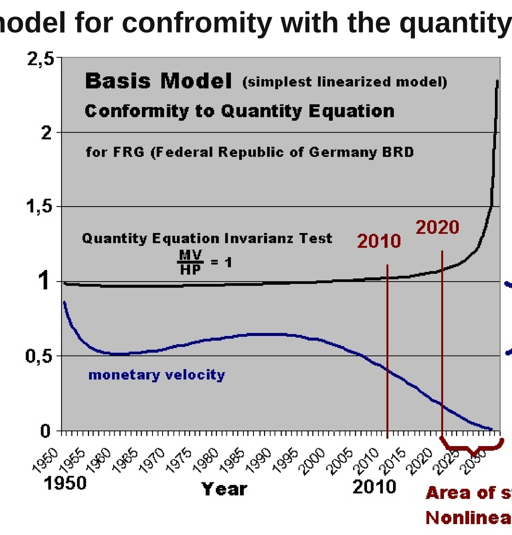


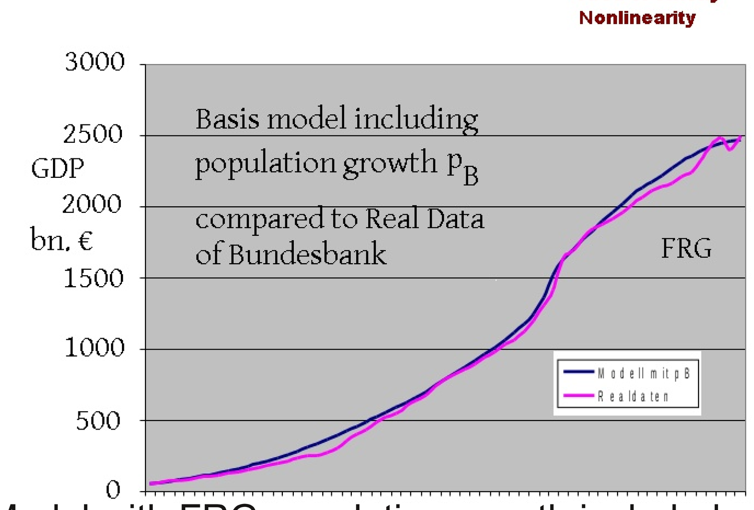
How the special theory works...

The complexity of theory can be reduced a lot by some simplifications and linearizing. Even the most simplified model but gives surprisingly good results. The accuracy but can be boosted to any desired precision if one incorporates good informations. This are especially the accurate official numbers as for the total amount of all assets, the fraction of Banks-own-business and to real economy, and the exact contribution to and from the international economies. This data are but not accurate available for every country. Mostly official data are fragmentary or of bad quality. The much easier handling of the special theory leads to a lot of new economic insights. Only some samples can be presented here for brevity.



Check the model for confromity with the quantity equation:

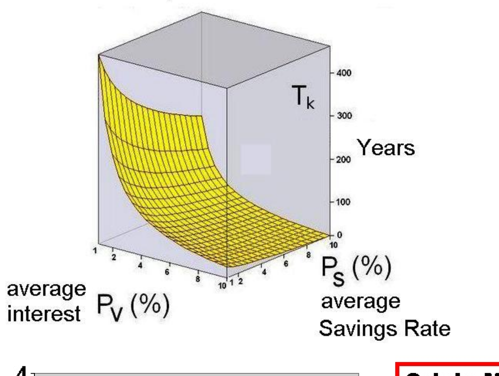




Model with FRG population growth included.
The strongest effect results from the inclusion of the German Democratic Republik in 1990.
The more parameters, the more accuracy.

Crisis Analysis

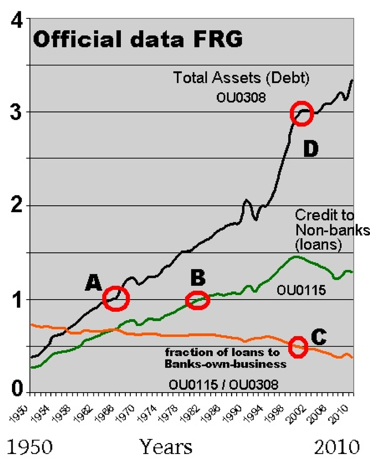
life time expectation for economies



(half-) Lifetimes of Economies can be calculated from different views. For debt-driven economies the time until crisis occurs depends on few parameters and are always between 50 and 100 years.

Typical at about 60 years.

Rule of Thumb: $T_k\approx\frac{3}{p_v}$



Crisis Markers:

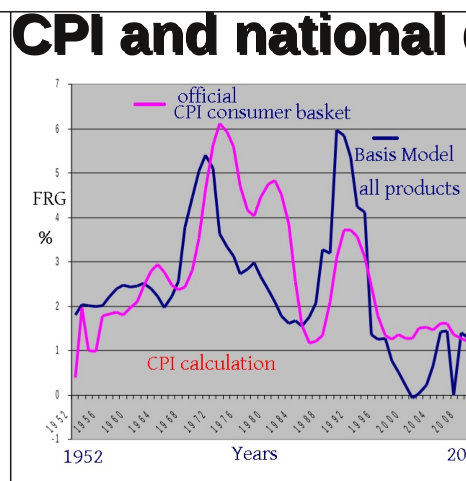
(A) (1967) Total Capital Coefficient K/Y exceeds 1 (100%) Economy changes from GDP-driven to Capital driven Economy

(B) (1982) Credits to Real Economy exceeds 100% of GDP Economy gets overloaded of debt

(C) (2001) Fractions of loans to Real Economy gets less than 50% of Banks Business total. => CRISIS

(D) (2001) Total Capital Coefficient K/Y exceeds 3 (> 300 %) => CRISIS

CPI and national debt



For the analytical core inflation I_c , we can therefore write:

$$I_c(t)=\frac{(\frac{\dot{Y}^2}{Y}+\frac{\dot{K}\dot{Y}^2}{Y^2})\cdot t+\dot{Y}+\ddot{K}}{Y+\dot{K}}$$

Or in terms of slip rates of growth

$$I_c(t)=p_W^2\cdot t+\frac{p_W Y+p_n^2 K}{Y+p_n K}$$

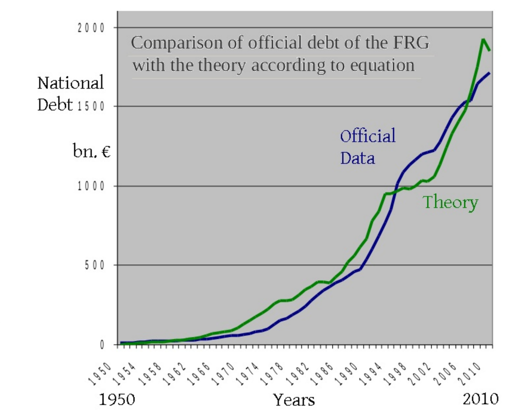
for the multi-year state budget balance is

$$\Delta S_f:=\sum_q^T V_q M_q^S-\sum_q^{T+1} V_q M_q^S\neq0$$

usually equal to zero,
but in most cases it is even negative:

$\Delta S_f<0$ deficient state
$\Delta S_f=0$ ideal state
$\Delta S_f>0$ profitable state

$$S_{SS}(T)=\sum_T p_S(T)\cdot Y(T)\cdot(1+p_A)^{T-T_0}$$



Substitutional Economy...

Macro- and Microeconomic Substitution Rules

The general substitution results from the relation

$$\text{macroec. demand}=\sum_i M_i V_i=\sum_j H_j P_j=\text{macroec. supply} \ .$$

Where we have seen in connection with the derivation of the law of Gossen after the *k-th* consumer choice H_k , we can now formulate the most general context of substitution:

$$\frac{\partial}{\partial H_k}\sum_{i,j}(M_i V_i-H_j P_j)=\frac{\partial}{\partial H_k}(M_k V_k)-P_k+\sum_{(i,j)-1}(M_i V_i-H_j P_j)$$

Because of the enormous diversity of economic products[54], ie $i,j\to\infty$, the last term provides the case of a particulate product decision for only one product

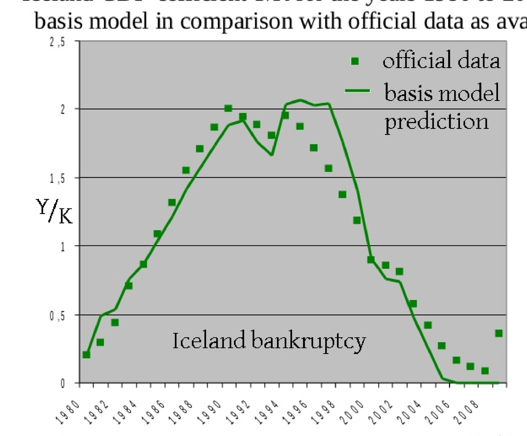
$$R_{VWL}:=\sum_{(i,j)-1}(M_i V_i-H_j P_j)\simeq0 \qquad (26.2)$$

fromwhich we get a zero contribution, since the removal of the kth product makes no appreciable change in the total.

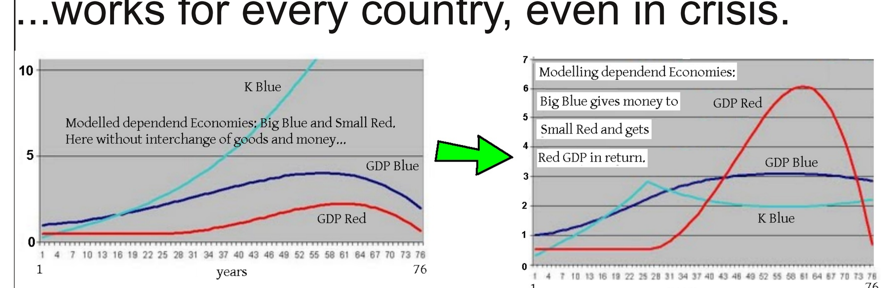
The relation (26.2) is exactly the deciding factor as to whether the treatment of a system as macroeconmical (open system) or a microeconomical problem (closed system) is necessary or possible. The economic balance $R_{VWL}\simeq0$ should disappear for microeconomical problems.

The World's Global Economy...

Iceland GDP-cefficient *Y/K* for the years 1980 to 2005, basis model in comparison with official data as available.



...works for every country, even in crisis.



...dependencies of economies can be handled

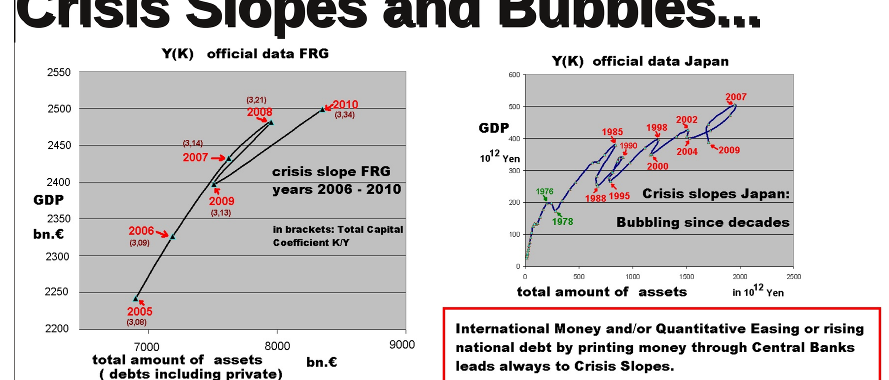
In principle this can be extended to the entire world economy by the following System of Differential Equations

$$\dot{Y}_i=\sum_j b_{0ij}+p_B Y_i-p_n K_i \quad\text{and}\quad \dot{K}_i=\sum_j a_{0ij}+p_S Y_i+p_n K_i$$

wherein the sizes of a_{0ij} and b_{0ij} considere the mutual interactions of the *i-th* with the *j-th* economy. b_{0ij} is then the net inflow or outflow of GDP without payment, a_{0ij} then is the net capital inflow or outflow without GDP trade-off. Inflow or outflow can be distinguished by the sign. The world economic system is then solved in principle by the maximal $2\cdot(i\times i)$ dimensional matrix

$$\dot{\vec{Y}}=\boldsymbol{b_0}\hat{\vec{Y}}+\boldsymbol{p_B}\vec{Y}_i-\boldsymbol{p_n}\vec{K}_i \quad\text{and}\quad \dot{\vec{K}}=\boldsymbol{a_0}\hat{\vec{K}}+\boldsymbol{p_S}\vec{Y}_i+\boldsymbol{p_n}\vec{K}_i$$

Crisis Slopes and Bubbles...



International Money and/or Quantitative Easing or rising national debt by printing money through Central Banks leads always to Crisis Slopes.

Slopes arrise because fresh money first accelerates growth, but shorthly later depresses GDP by its interest pressure.

Flaws of classical models...

FIRST: Not to include private assets / debt into the calculation!

SECOND: over-simplificated models, mostly not using the appropriate mathematical gadgets.